Sequential probability ratio tests based on grouped observations

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Abstract - This paper deals with sequential likelihood ratio tests based on grouped observations. It is demonstrated that the method of conjugated parameter pairs known from the non-grouped case can be extended to the grouped case obtaining Waldlike approximations for the OC- and ASN-function. For near hypotheses so-called F -optimal groupings are recommended. As example an SPRT based on grouped observations for the parameter of an exponentially distributed random variable is considered.¹

Index Terms - Hypotheses testing, probability ratio test, classified observations, grouped observations, sequential test, sequential analysis.

1 INTRODUCTION

Let $\{X_i\}_{i=1}^{\infty}$ be a sequence of i.i.d. random variables with a density function $f_{\theta}(x)$, $\theta \in \Theta$, with respect to some measure μ . Our aim is to discriminate between two simple hypotheses

$$H_0 : \theta = \theta_0 \quad \text{and} \quad H_1 : \theta = \theta_1, \quad \theta_0 \neq \theta_1, \quad (1)$$

by means of a sequential probability ratio test (SPRT).

In this context we suppose, that the random variables $\{X_i\}_{i=1}^{\infty}$ can be observed only in a restricted manner as follows. Let $\mathcal{G}$ be a partition of the domain $\mathcal{X}$ of the random variables $\{X_i\}_{i=1}^{\infty}$ in disjoint subsets $\mathcal{X}_1, \dots, \mathcal{X}_m$, $m \geq 2$, such that on each observation stage i , $i = 1, 2, \dots$, instead of X_i only a random variable $X_i^{\mathcal{G}}$ can be observed defined by

$$X_i^{\mathcal{G}} = k \quad \Longleftrightarrow \quad X_i \in \mathcal{X}_k, \quad k = 1, \dots, m,$$

with

$$p_{\theta}^{\mathcal{G}}(k) = P_{\theta}(X_i^{\mathcal{G}} = k) = P_{\theta}(X_i \in \mathcal{X}_k) > 0$$

$\theta \in \Theta$. That means, instead of a special measured value we observe only a corresponding group number and we have a so-called grouped or classified observation scheme. The partition $\mathcal{G}$ of $\mathcal{X}$ is called a grouping.

In the following we consider a WALD SPRT for the hypotheses (1) based on observations of the random variables

$\{X_i^{\mathcal{G}}\}_{i=1}^{\infty}$. This test is defined as follows (see, e.g., [5], [3] or [4]). Let

$$L_{n, \theta_0, \theta_1}^{\mathcal{G}} = \prod_{i=1}^n \frac{p_{\theta_1}^{\mathcal{G}}(X_i^{\mathcal{G}})}{p_{\theta_0}^{\mathcal{G}}(X_i^{\mathcal{G}})}$$

be for $n = 1, 2, \dots$ the likelihood ratio of sample $\vec{X}_n^{\mathcal{G}} = (X_1^{\mathcal{G}}, \dots, X_n^{\mathcal{G}})$, then to given stopping bounds $0 < B < 1 < A < \infty$ the sample size $N_{\mathcal{G}}$ and the termination rule $\delta_{\mathcal{G}}$ of a WALD SPRT for the hypotheses (1) are defined as follows:

$$N_{\mathcal{G}} = \inf\{n \geq 1 : L_{n, \theta_0, \theta_1}^{\mathcal{G}} \notin (B, A)\}$$

and

$$\delta_{\mathcal{G}} = 1_{\{L_{N_{\mathcal{G}}, \theta_0, \theta_1}^{\mathcal{G}} \leq B\}}.$$

That means, we continue the observations for $n = 1, 2, \dots$ as long as the critical inequality $B < L_{n, \theta_0, \theta_1}^{\mathcal{G}} < A$ holds. If on observation stage n for the first time $L_{n, \theta_0, \theta_1}^{\mathcal{G}} \notin (B, A)$ and if then $L_{n, \theta_0, \theta_1}^{\mathcal{G}} \leq B$ or $L_{n, \theta_0, \theta_1}^{\mathcal{G}} \geq A$ holds we accept the hypothesis H_0 or H_1 , respectively. We denote this SPRT by $S_{\mathcal{G}}(B, A)$.

The most important characteristics for evaluation of the statistical properties of our test are the operating characteristic function (OC-function) $Q_{\mathcal{G}}(\theta) = E_{\theta} \delta_{\mathcal{G}}$, $\theta \in \Theta$, and the average sample number function (ASN-function) $E_{\theta} N_{\mathcal{G}}$, $\theta \in \Theta$.

If $P_{\theta}(L_{1, \theta_0, \theta_1}^{\mathcal{G}} = 1) < 1$ then we have $P_{\theta}(N_{\mathcal{G}} < \infty) = 1$ and $E_{\theta} N_{\mathcal{G}} < \infty$. Moreover, the WALD-WOLFOWITZ-Theorem holds. That means, the test $S_{\mathcal{G}}(B, A)$ minimises the average sample number function for $\theta = \theta_0$ and $\theta = \theta_1$ among all tests whose error probabilities are not greater than the error probabilities of WALD's SPRT at $\theta = \theta_0$ and $\theta = \theta_1$.

The general problem of WALD's SPRT consists in the computation of its characteristics, e.g., the OC-function or the ASN-function. This especially holds for the grouped case considered here. We will demonstrate that the so-called method of conjugated parameter pairs known from the non-grouped case (see [3]) can be extended to the grouped case obtaining Waldlike approximations for the OC- and ASN-function. Moreover we will discuss some possibilities for the determination of optimal groupings. In this context the FISHER information and the so-called F -optimal groupings will play an important part. As example we consider an SPRT based on grouped observations for the parameter of an exponential distribution and present corresponding F -optimal groupings.

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2 THE WALD APPROXIMATIONS

The OC- and ASN-function of test $S_G(B, A)$ can be computed approximately in sense of the so-called WALD approximations by means of conjugated parameter pairs as follows [3].

DEFINITION. Two parameter pairs (θ', θ'') and $(\theta_0, \theta_1) \in \Theta \times \Theta$ are said to be *conjugated*, if a real number h , $h \neq 0$, exists, such that

$$L_{n, \theta', \theta''}^G = (L_{n, \theta_0, \theta_1}^G)^h, \quad n = 1, 2, \dots,$$

holds. We write: $(\theta', \theta'') \stackrel{h}{\sim} (\theta_0, \theta_1)$.

If $(\theta', \theta'') \stackrel{h}{\sim} (\theta_0, \theta_1)$ the OC-function $Q_G(\theta)$ and the power function $M_G(\theta) = E_\theta(1 - \delta_G)$, $\theta \in \Theta$, of test $S_G(B, A)$ satisfy the relations

$$\frac{Q_G(\theta'')}{Q_G(\theta')} = E_{\theta'} \left((L_{N, \theta_0, \theta_1}^G)^h | H_0 \text{ is acc.} \right) \leq B^h \quad (2)$$

and

$$\frac{M_G(\theta'')}{M_G(\theta')} = E_{\theta'} \left((L_{N, \theta_0, \theta_1}^G)^h | H_1 \text{ is acc.} \right) \geq A^h, \quad (3)$$

where in case of

$$\begin{aligned} P_{\theta'}(L_{N, \theta_0, \theta_1}^G = B | H_0 \text{ is accepted}) \\ = P_{\theta'}(L_{N, \theta_0, \theta_1}^G = A | H_1 \text{ is accepted}) = 1 \end{aligned} \quad (4)$$

the equals signs hold. We remark, that in case of $P_\theta(N_G < \infty) = 1$ (closed test) moreover $M_G(\theta) = 1 - Q_G(\theta)$ holds. A sufficient condition for closeness is, for instance, $P_\theta(L_{1, \theta_0, \theta_1}^G = 1) < 1$.

2.1 THE OC-FUNCTION

For a closed test $S_G(B, A)$ we get under the condition (4) and $(\theta', \theta'') \stackrel{h}{\sim} (\theta_0, \theta_1)$ by (2) and (3) for the OC-function

$$Q_G(\theta') = Q_G^*(\theta') = \frac{A^h - 1}{A^h - B^h}$$

and

$$Q_G(\theta'') = Q_G^*(\theta'') = B^h Q_G^*(\theta').$$

If condition (4) holds approximately, that means the excess over the stopping bounds is negligible, then we have $Q_G(\theta') \approx Q_G^*(\theta')$ and $Q_G(\theta'') \approx Q_G^*(\theta'') = B^h Q_G^*(\theta')$. This are the famous WALD approximations for the OC-function. If to given θ' an $h \neq 0$ and $\theta'' \neq \theta'$ do not exist such that $(\theta', \theta'') \stackrel{h}{\sim} (\theta_0, \theta_1)$, e.g., in case of $E_{\theta'} Z_{1, \theta_0, \theta_1}^G = E_{\theta'} \ln L_{1, \theta_0, \theta_1}^G = 0$, we can extend the Wald approximation for the OC-function by $Q_G(\theta') \approx Q_G^*(\theta') = \ln A / (\ln A - \ln B)$ for $E_{\theta'} Z_{1, \theta_0, \theta_1}^G = 0$.

2.2 THE STOPPING BOUNDS

Under the conditions (4) we obtain a test $S_G(B, A)$ at size (α, β) , $0 < \alpha, \beta < 1$, $\alpha + \beta < 1$, that means $Q_G(\theta_0) = 1 - \alpha$ and $Q_G(\theta_1) = \beta$, if the stopping bounds B and A satisfy the condition

$$B = B^* = \frac{1 - \beta}{\alpha} \quad \text{and} \quad A = A^* = \frac{\beta}{1 - \alpha}. \quad (5)$$

The values B^* and A^* are the so-called WALD approximations for the stopping bounds.

A sufficient condition for an admissible test for the hypotheses (1) at size (α, β) is $B = \beta$ and $A = 1/\alpha$. Then we have $Q_G(\theta_0) \geq 1 - \alpha$ and $Q_G(\theta_1) \leq \beta$.

2.3 THE ASN-FUNCTION

By means of the moment equation $E_\theta Z_{N, \theta_0, \theta_1}^G = E_\theta N_G \cdot E_\theta Z_{1, \theta_0, \theta_1}^G$ which holds for our tests if, e.g., $P_\theta(L_{1, \theta_0, \theta_1}^G = 1) < 1$ we get in case of $E_\theta Z_{1, \theta_0, \theta_1}^G \neq 0$ for the average sample number

$$\begin{aligned} E_\theta N_G &= (E_\theta(Z_{N, \theta_0, \theta_1}^G | H_0 \text{ is acc.}) Q_G(\theta) \\ &+ E_\theta(Z_{N, \theta_0, \theta_1}^G | H_1 \text{ is acc.})(1 - Q_G(\theta)) / E_\theta Z_{1, \theta_0, \theta_1}^G. \end{aligned}$$

If we again assume that condition (4) holds approximately we obtain the so-called WALD approximation $E_\theta^* N_G$ for the average sample number $E_\theta N_G$:

$$E_\theta N_G \approx E_\theta^* N_G = \frac{\ln B Q_G^*(\theta) + \ln A (1 - Q_G^*(\theta))}{E_\theta Z_{1, \theta_0, \theta_1}^G}. \quad (6)$$

In case of $E_\theta Z_{1, \theta_0, \theta_1}^G = 0$ we get by means of the moment equation $E_\theta (Z_{N, \theta_0, \theta_1}^G)^2 = E_\theta N_G \cdot E_\theta (Z_{1, \theta_0, \theta_1}^G)^2$ analogously the approximation $E_\theta N_G \approx E_\theta^* N_G = -\ln B \ln A / E_\theta (Z_{1, \theta_0, \theta_1}^G)^2$.

2.4 CONJUGATED PARAMETER PAIRS

According to our definition of conjugated parameter pairs we have in the i.i.d. case the following criterion. It holds $(\theta', \theta'') \stackrel{h}{\sim} (\theta_0, \theta_1)$ if to a given parameter value θ' a real number $h \neq 0$ and a parameter value $\theta'' \neq \theta'$ exist such that

$$\frac{p_{\theta''}^G(x)}{p_{\theta'}^G(x)} = \left(\frac{p_{\theta_1}^G(x)}{p_{\theta_0}^G(x)} \right)^h \quad (7)$$

holds for $x \in \{1, \dots, m\}$. Hence, a necessary existence condition for conjugated parameter pairs is, that the function $q_{\theta'}^G(x)$ defined by

$$q_{\theta'}^G(x) = \left(\frac{p_{\theta_1}^G(x)}{p_{\theta_0}^G(x)} \right)^h p_{\theta'}^G(x), \quad x \in \{1, \dots, m\},$$

is a probability mass function. Because of $q_{\theta'}^G(x) \geq 0$ for $x \in \{1, \dots, m\}$ we can compute a value h , $-\infty < h < \infty$,

such that

$$\begin{aligned}\varphi_{\theta'}(h) &= \sum_{x=1}^m q_{\theta'}^{\mathcal{G}}(x) = \sum_{x=1}^m \left(\frac{p_{\theta_1}^{\mathcal{G}}(x)}{p_{\theta_0}^{\mathcal{G}}(x)} \right)^h p_{\theta'}^{\mathcal{G}}(x) \\ &= E_{\theta'} e^{h Z_{1,\theta_0,\theta_1}^{\mathcal{G}}} = 1\end{aligned}$$

holds. The function $\varphi_{\theta'}(h)$ is as function of h , $-\infty < h < \infty$, the moment-generating function of the random variable $Z_{1,\theta_0,\theta_1}^{\mathcal{G}} = \ln L_{1,\theta_0,\theta_1}^{\mathcal{G}}$. It holds $\varphi_{\theta'}(0) = 1$, $\lim_{h \rightarrow \pm\infty} \varphi_{\theta'}(h) = \infty$, $\varphi_{\theta'}'(0) = E_{\theta'} Z_{1,\theta_0,\theta_1}^{\mathcal{G}}$ as well as $\varphi_{\theta'}''(h) = E_{\theta'} (Z_{1,\theta_0,\theta_1}^{\mathcal{G}})^2 e^{h Z_{1,\theta_0,\theta_1}^{\mathcal{G}}} > 0$. This means that $\varphi_{\theta'}(h)$ is a convex function in h . Hence, we have in case of $E_{\theta'} Z_{1,\theta_0,\theta_1}^{\mathcal{G}} < 0$ and $E_{\theta'} Z_{1,\theta_0,\theta_1}^{\mathcal{G}} > 0$ beside the trivial solution $h = 0$ of equation $\varphi_{\theta'}(h) = 1$ always an unique solution $h > 0$ and $h < 0$, respectively.

The case $m = 2$: In this case our test becomes an SPRT for discriminating between two probabilities. Then in case of $E_{\theta'} Z_{1,\theta_0,\theta_1}^{\mathcal{G}} \neq 0$ beside the solution $h \neq 0$ of $\varphi_{\theta'}(h) = 1$ always a parameter value $\theta'' \neq \theta'$ exists such that condition (7) holds for $x \in \{1, 2\}$. This implies $(\theta', \theta'') \stackrel{h}{\sim} (\theta_0, \theta_1)$ and we obtain the usual WALD approximations for the OC- and ASN-function.

The case $m > 2$: Examples show that as a rule to a given solution $h \neq 0$ of $\varphi_{\theta'}(h) = 1$ it does not exist a parameter value θ'' such that $p_{\theta''}^{\mathcal{G}}(x) = q_{\theta'}^{\mathcal{G}}(x)$, $x \in \{1, \dots, m\}$. However, we can find always a value θ'' such that this relation holds approximately. Hence we have then

$$\frac{p_{\theta''}^{\mathcal{G}}(x)}{p_{\theta'}^{\mathcal{G}}(x)} \approx \left(\frac{p_{\theta_1}^{\mathcal{G}}(x)}{p_{\theta_0}^{\mathcal{G}}(x)} \right)^h, \quad x \in \{1, \dots, m\},$$

and in sense of this approximation we can compute corresponding modified WALD approximations for the OC- and ASN-function if we use now the left-hand approximation for the OC-function in (2.1)

$$Q_{\mathcal{G}}(\theta') \approx Q_{\mathcal{G}}^*(\theta') = \frac{A^h - 1}{A^h - B^h}.$$

An explicit determination of the parameter value θ'' is not necessary then.

3 OPTIMAL GROUPINGS

The WALD approximations for the ASN-function provide hints how a grouping does influence to the average sample size. While the numerator in (6) is independent on the grouping $\mathcal{G}$ the denominator depends on $\mathcal{G}$ via the expectation value $E_{\theta} Z_{1,\theta_0,\theta_1}^{\mathcal{G}}$. This dependence can be used optimising the average sample number by means of an appropriate grouping for a given parameter value θ .

Especially, we get for our test corresponding small average sample sizes for θ_0 , θ_1 or θ^* in sense of the WALD approximations if a grouping $\mathcal{G}$ is chosen which maximises $|E_{\theta_0} Z_{1,\theta_0,\theta_1}^{\mathcal{G}}|$, $E_{\theta_1} Z_{1,\theta_0,\theta_1}^{\mathcal{G}}$ or, in case of $E_{\theta^*} Z_{1,\theta_0,\theta_1}^{\mathcal{G}} = 0$, the

expectation value $E_{\theta^*} (Z_{1,\theta_0,\theta_1}^{\mathcal{G}})^2$ with respect to $\mathcal{G}$, respectively.

An interesting case are near hypotheses: If $\Delta\theta = |\theta_1 - \theta_0|$ is small, then it can be shown that $E_{\theta_0} Z_{1,\theta_0,\theta_1}^{\mathcal{G}} = -\frac{1}{2} I_F^{\mathcal{G}}(\theta_0)(\Delta\theta)^2$, $E_{\theta^*} (Z_{1,\theta_0,\theta_1}^{\mathcal{G}})^2 = I_F^{\mathcal{G}}(\theta^*)(\Delta\theta)^2$ and $E_{\theta_1} Z_{1,\theta_0,\theta_1}^{\mathcal{G}} = \frac{1}{2} I_F^{\mathcal{G}}(\theta_1)(\Delta\theta)^2$ for $\Delta\theta \rightarrow 0$ holds, where

$$I_F^{\mathcal{G}}(\theta) = E_{\theta} \left(\frac{\partial \ln p_{\theta}^{\mathcal{G}}(X_1^{\mathcal{G}})}{\partial \theta} \right)^2$$

denotes the FISHER information of a single observation of random variable $X_1^{\mathcal{G}}$ depending on the parameter value θ . This underlines the importance of the FISHER information with respect to optimal groupings in this context.

DEFINITION. Let $\mathcal{G} = \{\mathcal{X}_1, \dots, \mathcal{X}_m\}$ be an interval grouping such that $\mathcal{X}_i = [x_{i-1}^*, x_i^*]$ for $i = 1, \dots, m$ and $\inf_{x \in \mathcal{X}} x = x_0^* < x_1^* < \dots < x_m^* = \sup_{x \in \mathcal{X}} x$, holds. An interval grouping $\mathcal{G}_0$ is said to be F -optimally for θ , if

$$I_F^{\mathcal{G}_0}(\theta) = \max_{x_1^*, \dots, x_{m-1}^*} I_F^{\mathcal{G}}(\theta)$$

holds.

The F -efficiency of such a grouping can be measured by the ratio $F_{\text{eff}}(\mathcal{G}, \theta) = I_F^{\mathcal{G}}(\theta)/I_F(\theta)$, where $I_F(\theta) = E_{\theta} \left(\frac{\partial \ln f_{\theta}(X)}{\partial \theta} \right)^2$ denotes the FISHER information of a non-grouped observation of the random variable X_1 . It holds $0 \leq F_{\text{eff}}(\mathcal{G}, \theta) \leq 1$. Numerical studies show that F -optimal groupings are quite robust with respect to their efficiencies against modifications of the group bounds. For instance, a simplification of the group bounds by moderate rounding does not lead to a significant loss of F -efficiency or discrimination information.

Table 1 F -optimal group bounds of interval groupings $\mathcal{G}_m$ for $\theta = 1$, $m = 2, \dots, 9$, and F -efficiencies $F_{\text{eff}}(\mathcal{G}_m, \theta)$ [%] - exponential distribution.

m	2	3	4	5
x_1^*	1.5936	1.0176	0.7540	0.6004
x_2^*		2.6112	1.7716	1.3545
x_3^*			3.3652	2.3720
x_4^*				3.9657
%	64.76	82.03	89.10	92.69

m	6	7	8	9
x_1^*	0.4993	0.4276	0.3739	0.3323
x_2^*	1.0997	0.9269	0.8015	0.7062
x_3^*	1.8538	1.5273	1.3008	1.1338
x_4^*	2.8714	2.2813	1.9012	1.6331
x_5^*	4.4650	3.2989	2.6553	2.2336
x_6^*		4.8925	3.6729	2.9876
x_7^*			5.2665	4.0052
x_8^*				5.5988
%	94.76	96.06	96.93	97.54

Figure 1 WALD approximations $Q_{\mathcal{G}_m}^*(\theta)$ of the OC-functions of tests $S_{\mathcal{G}_m}(B^*, A^*)$ for $m = 2, \dots, 9$ and $Q^*(\theta)$ of the non-grouped test $S(B^*, A^*)$.

Figure 2 WALD approximations $E_{\theta}^* N_{\mathcal{G}_m}$ of the ASN-functions of tests $S_{\mathcal{G}_m}(B^*, A^*)$, $m = 2, \dots, 9$ and $E_{\theta}^* N$ of the non-grouped test $S(B^*, A^*)$ (bold).

4 EXAMPLE

Let $\{X_i\}_{i=1}^{\infty}$ be independent exponentially distributed random variables with density function

$$f_{\theta}(x) = \begin{cases} \theta e^{-\theta x} & \text{for } x \geq 0 \\ 0 & \text{for } x < 0, \end{cases}$$

$0 < \theta < \infty$. Our aim is to discriminate between the hypotheses $H_0 : \theta = \theta_0$ and $H_1 : \theta = \theta_1$, $\theta_0 < \theta_1$, where $0 < \theta_0 < 1 < \theta_1 < \infty$ and $E_{\theta} Z_{1, \theta_0, \theta_1} = \ln \theta_1 / \theta_0 - (\theta_1 - \theta_0) / \theta = 0$ for $\theta = 1$ holds. This side condition is no restriction since the parameter θ is a scale parameter here and other simple hypotheses can be reduced to this one by an appropriate transformation of the random variables $\{X_i\}_{i=1}^{\infty}$.

Since $E_{\theta} Z_{1, \theta_0, \theta_1} < 0$ and $E_{\theta} Z_{1, \theta_0, \theta_1} > 0$ for $\theta < 1$ and $\theta > 1$, respectively, the test should prefer the hypothesis H_0 for the first case and the hypothesis H_1 for the second one. That means, the test should be most selective for parameter values in the neighbourhood of $\theta = 1$. This can be reached by means of F -optimal groupings for $\theta = 1$.

Table 1 presents the corresponding F -optimal group bounds $x_1^*, \dots, x_{m-1}^*$ of interval groupings $\mathcal{G}_m$ for $\theta = 1$ and $m = 2, \dots, 10$ as well as the reached relative efficiencies (in percent) $F_{\text{eff}}(\mathcal{G}_m, \theta) = I_F(\mathcal{G}_m, \theta) / I_F(\theta)$, $I_F(\theta) = E_{\theta}(\partial \ln f_{\theta}(x) / \partial \theta)^2 = 1 / \theta^2$ for $\theta = 1$.

We now consider the special hypotheses $H_0 : \theta_0 = 0.85$ and $H_1 : \theta_1 = 1.166687$. Then we have $E_{\theta} Z_{1, \theta_0, \theta_1} = 0$ for $\theta = 1$. Let $\alpha = 0.05$ and $\beta = 0.05$ be the given risks of an error of first and second kind, respectively. Then we get by (5) the following WALD approximations for the stopping bounds B and A : $B^* = 0.052632$ and $A^* = 19$.

Figure 1 shows the WALD approximations of the OC-functions obtained by the method of conjugated parameter pairs of Section 2.4 for the tests $S_{\mathcal{G}_m}(B^*, A^*)$, $m = 2, \dots, 10$, based on the F -optimal interval groupings $\mathcal{G}_m$ of Table 1. We see that grouping has only a slight influence on the WALD approximation of the OC-function. That means, the OC-function of WALD's SPRT remains almost unaltered if we switch over from non-grouped to grouped observations.

Figure 2 presents the corresponding WALD approximations of the ASN-functions of the tests $S_{\mathcal{G}_m}(B^*, A^*)$, $m = 2, \dots, 10$, as well as the WALD approximation of the ASN-function $E_{\theta}^* N$ for the non-grouped test $S(B^*, A^*)$ (bold line). Here we can see how grouped observations increase the WALD approximations of the ASN-functions of our tests depending on the F -efficiency of a grouping or the number of groups, respectively.

5 CONCLUSIONS

The method of conjugated parameter pairs is an effective method obtaining Waldlike approximations for the OC- and ASN-function for sequential likelihood ratio tests based on grouped observations. With respect to their high efficiencies F -optimal groupings are recommended.

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