Energy-sensitive process chain optimization on the example of forging

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Abstract

We present a mathematical optimization model for the determination of an energy-minimal process chain, given a metro map describing the single process chain variants and a description of the processes themselves. This approach is applied to a small process chain example that includes a forging process. For some of the processes several parameter settings can be used. That leads to process variants in the optimization model that differ in the required input temperatures. After the calculation of the optimal process chain for the considered example we extend our mathematical model in such a way that uncertainties concerning the values of the energy demands can be taken into account.

Keywords:

energy efficiency, process chain design, forging, integer programming, robust optimization

1 Introduction

Due to the scarcity of resources and increasing environmental awareness there is a need for energy-sensitive solution approaches in production. One way to reduce the energy demand in production is to improve or substitute single processes. In this paper we present a mathematical optimization model that, in addition to this, allows to determine an energy-minimal process chain on basis of a given metro map [1] and a description of the process variants via the input-throughput-output model [2]. The final process chain satisfies given requirements concerning dependencies between the

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single processes. These dependencies between the processes may lead to solutions that take advantage of combining certain processes, to the effect that some of these processes are not optimal on there own while their combination is. This could, *e. g.*, be the case for two processes with the second process using the exhaust heat of the first one.

The paper is structured as follows. In Section 2 the mathematical model is introduced. This is based on a directed graph G, that is closely related to the metro map and represents several process chain variants. We describe in detail how the graph G can be derived from the metro map and the information on the processes. A main task is to create process variants for each process that may differ in the method of process, *i. e.* the variants to realize the process, but also in the employed machines and tools as well as in the parameters used for the process. We formulate this as a binary *minimum cost flow problem* with *coupling constraints*. The latter ensure that dependencies between the processes resp. process variants are taken into account, for example if two processes are mutually exclusive. These models can be solved using standard solvers for integer programming.

The method presented in Section 2 is applied to a small example. It starts with a metal bar that is cut to length and may then be heated to one of five different temperatures. After that the forging and deflashing processes are performed. We describe the technical details for this in Section 3. In Section 4 we present the mathematical model for this illustrative example and calculate the optimal value. During optimization we have to ensure via coupling constraints that the input and output temperatures of the workpiece fit together for the different processes.

In practice, the actual energy demands of the processes may differ significantly from the estimated values employed in the optimization model. The correct energy demands of the processes differ in comparison to the values used in optimization, for example due to measurement errors. In Section 5 we extend the mathematical model so that uncertainties concerning the energy demands are respected. Assuming normally distributed energy demands we add so called *chance constraints*. The effect for the solution of different covariance matrices is demonstrated.

2 Mathematical model for process chain optimization

In this section we present a general way to describe the various process chains in an optimization model and to find an optimal one among them. The mathematical model is based on the metro map for the description of process chains that was developed in eniPROD [1], see figures 1, 2 and 3 for various unrelated examples. In such a map processes are represented by the “stations” S and a process that may succeed another process is connected by a line. The colors of the lines between the stations (in a real metro map a part of a metro line) may encode that there are different materials or semi-finished products at the beginning of the production (Figure 1) but at most one of them may be used or that there is a certain number of starting materials (Figure 2), for example if one of the processes is a joining process. In [1] the metro map is visualized with undirected edges, but it is better to think of them in a directed way. So the lines are directed in all our figures.

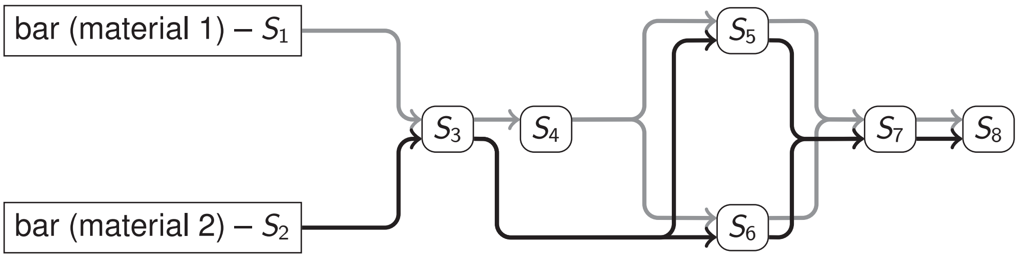

Figure 1: Metro map for a small example with two possible materials

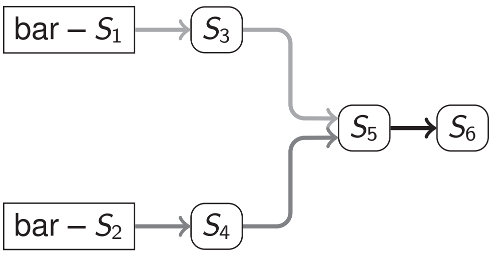

Figure 2: Metro map for a small example with joining

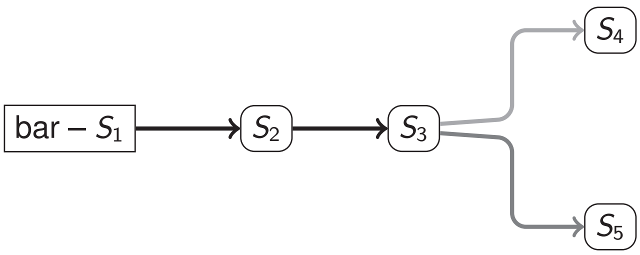

Figure 3: A metro map for a small example with a process (S_3) that enlarges the number of important components from one to two, *i. e.*, after that both components are processed independently

Based on the metro map we develop an integer programming model with coupling constraints for finding a best process chain with respect to the objective function. In our investigations we only consider the energy demand of the whole process chain but it is also possible to take costs and/or times into account. The solution of the optimization model then corresponds to an optimal process chain.

In the following we construct a directed graph $G = (N, P)$ with node set N and set of directed arcs P . This graph is closely related to the corresponding metro map with the roles of nodes and arcs interchanged in the following sense. While in the metro map nodes refer to stations and arcs to lines, in the new graph a node $n \in N$ refers to the state after a certain process has finished. An arc $p = (p_1, p_2) \in P$ demonstrates that after the process corresponding to p_1 is finished there is the possibility to attain the state corresponding to p_2. So $p \in P$ complies with a fully specified process or process variant with specific input, throughput and output parameters. As we will see below there may be several arcs from a node $n_i \in N$ to a node $n_j \in N$, $n_i \neq n_j$. This enables us to describe on the one hand several methods of process and on the other hand for example different input parameters like starting materials. This case is visualized in Figure 4. Graph G contains two arcs from n_1 to n_2 (here the two lines [black and gray] can be treated in only one graph with the same nodes; we later have to ensure via coupling constraints that the determined process chain only uses arcs that correspond to one color).

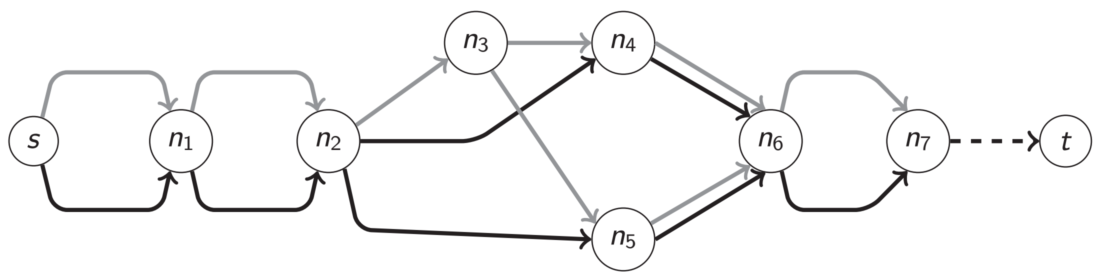

Figure 4: Graph G for the metro map in Figure 1 assuming that there is exactly one process variant for each process and each of the two materials, for each process independent of the previous processes (otherwise there may be more than one arc from one node to another).

Besides the processes themselves we can consider common resources like cooling, certain machines or generators. We collect these in set R. The consideration of resources separate from the processes themselves allows us, *e. g.*, to restrict the time spent on a certain resource or to count the base load of a resource only once if it is used at all.

In the following we specify additional nodes and arcs that are introduced for the treatment of the starting materials and of processes that change the number of considered components. For the sake of simplicity we assume that if there are several components before a process we know exactly which components are joined to one component. Similarly, for splitting processes we know the number and kind of new components. We changed the roles of nodes and arcs in comparison to the metro map because we will assign the energy demands of the processes to the arcs resp. to the variables of the arcs (there will be a variable for each arc). Because the starting materials or components should be considered, too, we introduce an artificial node s that is connected to all nodes that represent the state after the starting material was taken but before a single process has started. In Figure 4 these are the two arcs from s to n_1 and in Figure 5 these are the arcs $(s, n_1), (s, n_2)$ (via the constraints we will ensure that in the first case only one of these arcs is used and in the second case that the energy demand of both components is considered). Moreover we add an artificial node t that corresponds to the finished product or products (if more than one product is built at the same time, *e. g.*, as the result of certain cutting processes). Each state $n \in N$ that describes that the last process in this part of the process chain has been performed is connected to t. In figures 4 and 5 we included only one arc (n_7, t) resp. (n_6, t). Because two products are built in the example in Figure 6 we added the arcs $(n_6, t), (n_7, t)$.

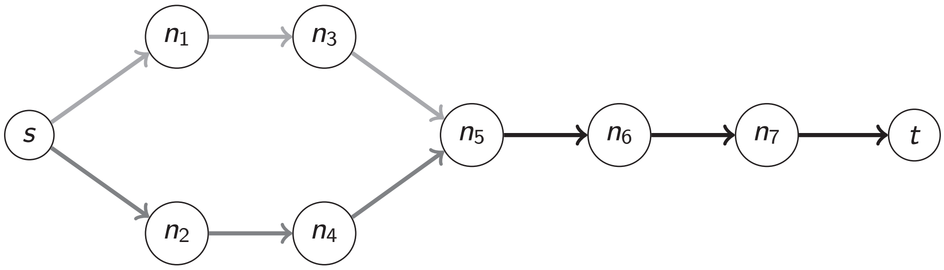

Figure 5: Graph G for the metro map in Figure 2 assuming that there is exactly one process variant for each process (otherwise there may be more than one arc from one node to another).

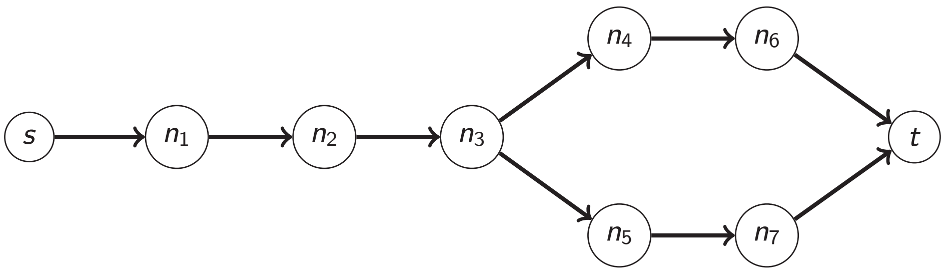

Figure 6: Graph G for the metro map in Figure 3 assuming that there is exactly one process variant for each process (otherwise there may be more than one arc from one node to another).

For a joining process with finished state $q \in N$, in which the number of considered components has been reduced (Figure 5), *e. g.* by joining, we add an artificial node $a_q \in N$. This new node represents the case that the single components are ready. It is connected via arcs (p, a_q) from all states $p \in N$ of components in which they can be joined. Furthermore we include (a, q) from $a_q \in P$ to the node $q \in P$. The example in Figure 5 depicts a joining process. For this we include the artificial node n_5. Here the joining process belongs to arc (n_5, n_6) with n_6 the state of the workpiece after joining.

For splitting processes with the states $q_i \in N, i = 1, \ldots, q_N$, of the q_N components after the process we add a new node a_p that represents that the respective process has been performed but we still look at this as one component. The splitting can be seen in the next nodes of the graph. Then we add arcs (p, a_p) from the previous states to the artificial node and connect via arcs $(a, q_i), i = 1, \ldots, q_N$, node $a_p \in P$. In the example depicted in Figure 6 the number of components is increased. We included the artificial node n_3 as well as the arc (n_2, n_3) representing the process and the arcs $(n_3, n_4), (n_3, n_5)$ that show the splitting in two components. The presented procedure for joining and splitting processes simplifies the assignment of energy demands to process variant as well as the description of input and output of these processes.

In the metro map a “process”, *i. e.* a single station in the map, can represent several process variants that may differ in the process parameters or in the used machines or tools. In the following we explain how the process variants are created. For the description of processes the input-throughput-output model is used [2]. The input includes the specification of the workpiece, *i. e.*, information on the geometry of the workpiece, on the material, on the temperature, on the precision and so on. For the throughput the machine(s) and the methods of process have to be specified. The specification of the methods of process comprises the used tools as well as information on the required resources, *e. g.*, cooling can but has not to be used for turning. Furthermore one gives details on the process parameters and restrictions on them. If the data was collected by measurements or via simulation, only discrete values are available for the parameters but if there is a detailed model for the process, *e. g.*, as in turning [2], parameters may be specified via feasible intervals and there may be restrictions that have to be fulfilled by the parameters (sometimes in dependence on the used machines, tools and resources). Finally, the throughput gives a calculation rule for the output and for the required resources, *e. g.*, amount of cooling medium or compressed air as well as the time used on resources $r \in R$.

The output describes the workpiece resulting from this process, especially changes in the geometry and changes of important properties, *e. g.*, if the temperature of the workpiece has been increased by a certain amount. Besides, the output includes information about the energy demand of each process variant, resp. the weighted total energy demand for the forms of energy (we are only interested in the energy demand for the forging example; if one takes further goals into account like time or costs, information on these values are needed here as well). For results from measurements the discrete data can be collected in a table. If the number of described variants in such a table is not too large we assign an arc to each variant. In Figure 1 the variants differ in the input, in particular on the material of the workpiece. In dependence of the input there may be changes in the throughput or the output. If the number of variants generated from the discrete values is too high, we can try to reduce this number in a first step. For example if the input of two process variants is the same and the geometry and the important properties in the output are the same, it suffices to include only the process variant with the smallest energy demand. This simplification is not possible if the required resources differ while at the same time the use of these resources is constrained by other restrictions.

If there is a mathematical model that describes throughput and output for a process for a given input with specified machines, tools and resources (the calculation of the energy demand may depend on the resources, for example turning with and without cooling) parameters are often allowed to be chosen from given intervals. Then we cannot test all parameter settings. In the case that for the specified values (input and parts of the throughput) the changes on the workpiece are the same for all parameter settings and there exist no further dependencies to following processes we can solve an optimization model and determine the best variant. But if there exist dependencies to subsequent processes we have to discretize the parameter values and determine the throughput and output for each of these variants. To get a rough feeling for the critical processes in the whole process chain it should be sufficient to create only few process variants in a first step, to solve the model and to refine the variants for critical processes in a second step before solving again.

Based on the graph G we build our optimization model. For this we introduce a binary variable $x_p \in \{0,1\}$ for all $p \in P$ that is one if and only if the corresponding process variant is used in the process chain. We identify each arc $p = (n,m)^i \in P$, $n, m \in N$, $i \in \mathbb{N}$, by its tail n, its head m and an index i, since there may be more than one arc from node n to node m. The variables have to fulfill the so called *flow conservation constraints*. These restrict the difference of the incoming and the outgoing arcs at each node $n \in N$. They read

$$\sum_{a=(n,m)^i \in P} x_a - \sum_{b=(m,n)^i \in P} x_b = c_n \qquad \text{for all } n \in N, \tag{1}$$

for a constant c_n depending on the graph G resp. the metro map as well as on $n \in N$, with $c_n = 0$ in many cases. In general, c_n, $n \in N$, describes the difference of the number of considered components before this state and the number of considered components after this state. For the example in Figure 4 c_{n_i} equals zero holds for all nodes n_i, $i \in \{1, \dots, 7\}$, because these nodes correspond to the state between two processes. If we reach one of these states there exists a further process that is performed afterwards or otherwise all variables summed up in (1) are zero. The situation changes for the artificial nodes s, t. For s, exactly one of the two materials should be used, so we get $\sum_{a=(s,m)^i \in P} x_a = 1$. Regarding node t we achieve that exactly one product is built by setting $c_t = -1$. This leads to $-\sum_{b=(m,t)^i \in P} x_b = -1$. The artificial node n_5 of the joining process in Figure 5 has $c_{n_5} = 1$ and we need $c_{n_3} = -1$ for the artificial node n_3 of the splitting process in Figure 6.

It highly depends on the actual process chain which couplings occur between the process variants and which of them have to be considered during optimization. We will indicate some important possibilities here. In the case that a process variant $p \in P$ forbids another process variant $q \in P$, for example if the output of p is no feasible input for q, we can add the constraint $x_p + x_q \leq 1$. In the example shown in Figure 4 we have to forbid by such constraints that solid black and solid gray arcs are used at the same time because these correspond to different materials, *e. g.*, $x_{(n_3,n_4)} + x_{(n_2,n_5)} \leq 1$. It may also happen that a process variant $p \in P$ can only take place if another process variant $q \in P$ takes place, too. An explanation for such a behavior are processes that use the exhaust heat of previous processes. Then the constraints $x_p \leq x_q$ have to be added. In the simplest case a process variant $p \in P$ is used if and only if process variant $q \in P$ is used, too. This leads to $x_p = x_q$.

In the next part we consider the resources. We introduce a binary variable $x_r \in \{0, 1\}$ for all resources $r \in R$ with the interpretation that x_r is one if and only if r is used in at least one of the process variants and zero otherwise. This implies new coupling constraints because $x_r = 0$ forbids all process variants that use this resource (see throughput). Similarly to the previous constraints we get $x_p \leq x_r$ for all processes $p \in P$ that use resource $r \in R$. In the case that for example the total time spent on a certain resource $r \in R$ is limited by K_r we get $\sum_{p \in P} r_p x_p \leq K_r$ with r_p time used by process variant $p \in P$ on r.

Finally, we define the objective function. Let e_p be the energy demand of process variant $p \in P$, resp. the energy value assigned to the starting material for arcs $p = (s, n)^i \in P$. All arcs $p = (n, t)^i \in P$ fulfill $e_p = 0$. The energy demand of a process with changing number of considered components is assigned to the arc from the introduced artificial node to the node representing the state after that process for a joining process and to the arc of the previous state to the artificial node for splitting processes. The coefficients of the other corresponding arcs are zero in both cases. Looking at the graph in Figure 5 the coefficient of the variable corresponding to arc (n_5, n_6) equals the energy demand of the joining process and the coefficients of the variables $x_{(n_3,n_5)}, x_{(n_4,n_5)}$ are zero. Furthermore let e_r be the energy demand for the base load of resource $r \in R$. Then the objective function reads

$$\sum_{p \in P} e_p x_p + \sum_{r \in R} e_r x_r \to \min . \tag{2}$$

3 Description of the process chain example

Improving the energy efficiency in forming processes resp. in corresponding process chains is possible via saving resources, a reduction of the working time or a reduction of the energy demands. As a practical example we choose a process chain of a common product of the automotive sector like a transmission shaft. Even though it is a rather short process chain, it contains several different kinds of operations. Each of these is influenced by several parameters that have to be analyzed and then combined to get an optimal process chain.

The first step is to prepare the part to be inserted in the forming dies. In our case it has a simple cylindrical shape with radius 30 mm and we cut it to length from a previous rolled rod of steel 20MnCr5, a typical material for that kind of component in the powertrain segment. After that, we heat up the component to prepare it for the deformation process. The steel which we selected can be forged in a temperature's range from about 800°C to 1200°C. We tested the five different temperatures 800°C, 900°C, 1000°C, 1100°C and 1200°C.

The part is heated up in an furnace, in our case a resistance furnace, in which the electricity, flowing in a resistance's system, heats them up following Joule's law. Under the same furnace conditions, the warming time depends on the required temperature; the higher the temperature the shorter is the warming time. At the moment, different kinds of measurements are performed on the test-furnace in the project PT of the cluster eniPROD to find the real energetic demand of this step in order to validate the numerical solutions by experimental results. In the example considered here it seems an easy choice to select the lowest possible forming temperature. But in general, looking only at the temperature does not offer us a complete insight on the forming process because we have to evaluate the mechanical properties of the finished part, which are strongly influenced by the forging temperature and have to consider the mechanical loads applied on the forming dies which directly affect the number of components produced within the tool's life.

After heating it up, the hot part is taken out of the furnace as fast as possible and inserted directly in the forming dies to be forged in order to reduce the heat loss of the workpiece and the energy dissipation via irradiation in the environment. These tools are preheated to the working temperature of approximately 150°C in order to avoid possible failures during the operation and too high thermal loads. The forming process is accomplished by means of a fly press, a particular kind of screw press in which the screw shaft driving the ram is actuated by a flywheel connected to a motor using a friction coupling. We assume that we can vary the forming force of the ram and the driving speed of the flywheel for the machine used. In impression-die forging the workpiece acquires the shape of the die cavities while it is being upset between the closing dies. Some of the material flows radially outward and forms a flash. The higher the workpiece temperature the lower are the required forces by the machine to form it and the lower the mechanical loads on the forming tools.

After that the workpiece is deflashed in order to prepare it for trimming and finishing operations which completes our process chain. (In our example we do not take the trimming and finishing operations into account.) This process is performed in another working-step on a press in which the overflowing material is cut from the part in one stroke by means of appropriate dies, which have a cutting contour of the same form of the predicted flash. In our case, for a matter of costs, we decided to calculate the energy demand of this process numerically instead of measuring it during the process. Numerical modeling has increasingly been applied to the energy-efficient design and optimization of hot forming processes. Using numerical simulation we were able to reduce the number of required practical experiments in a laboratory significantly. We selected several reasonable working conditions, varying workpiece's temperature and machine properties, and tested these in simulation, using the finite element software FORGE2011 [3]. Figure 7 shows the energy demands for the three varying parameters temperature, flash thickness and ram speed. As result of these analysis we gained first insights of the necessary forming energy which will be soon compared with the result of the forthcoming experimental campaign in the laboratory of the Fraunhofer IWU.

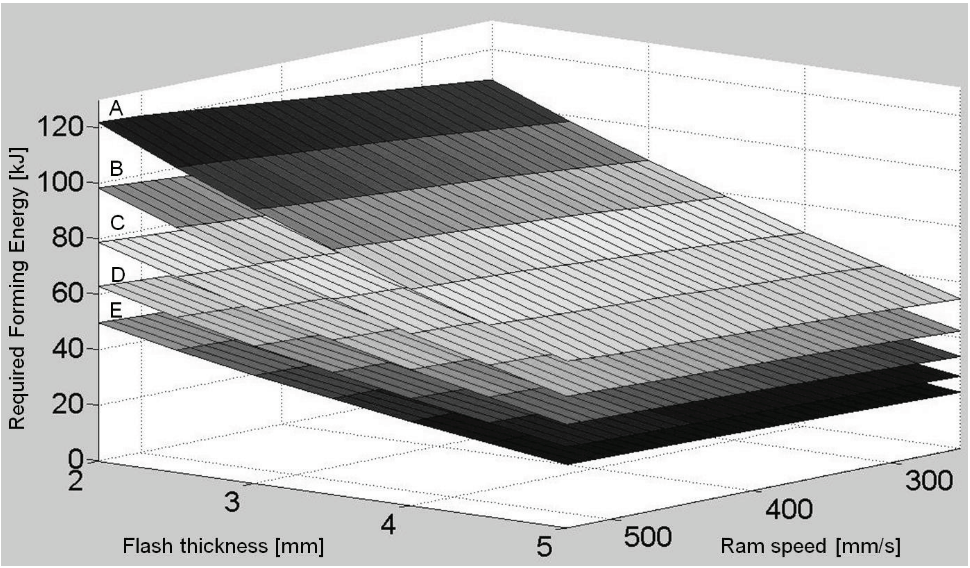

Figure 7: Energy demand of the forging process in different working conditions obtained via simulation using the temperatures 800°C (A), 900°C (B), 1000°C (C), 1100°C (D), 1200°C (E), flash thicknesses 2 mm, 3 mm, 4 mm, 5 mm and ram speeds 250 $\frac{mm}{s}$, 350 $\frac{mm}{s}$, 450 $\frac{mm}{s}$, 500 $\frac{mm}{s}$ and 530 $\frac{mm}{s}$

4 Optimization model for the process chain example

In this section we derive the optimization model for the process chain described in the previous section. The metro map of this process chain is visualized in Figure 8. Apart from the number of arcs this leads to the structure of the graph G presented in Figure 9. The example graph $G_e = (N_e, P_e)$ consists of the nodes s, t and the nodes

Figure 8: Metro map for the forging example

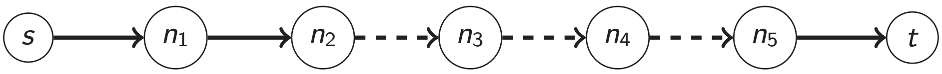

Figure 9: Structure of graph G_e for the metro map in Figure 8: There might be several arcs between two nodes. Indeed, there is more than one arc for dashed connections.

n_i, $i = 1, \ldots, 5$. Node n_1 represents the state that all materials (the bar) are available. The state that the bar has the correct length is assigned to node n_2. Node n_3 says the workpiece is heated and n_4 the workpiece is forged. Finally, n_5 represents the state that the deflashing process is over. As described in Section 3 we use a fixed material for the whole process chain. So we only use one arc (s, n_1) and assign the energy demand to this resp. the corresponding variable. On the next arc the metal bar is cut to the correct length. Based on the investigations in [2] there is a model that describes the energy demand of this process. The parameters that can be varied are the cutting speed v_c and the feed f. Since we assume that the choice of v_c, f does not influence the following processes we can determine the optimal parameter setting for a given material and a fixed machine and tool. The theoretical formula for the energy demand in J [2, 4, 5] reads

$$e_{cut} = W_{th} \cdot V = \frac{k_{c1,1}}{1000} \cdot K_{st} \cdot \left(\frac{100 \frac{\text{m}}{\text{min}}}{v_c} \right)^{m_v} \cdot \left(\frac{h_0}{f \cdot \sin(\kappa)} \right)^m \cdot a_p \cdot \pi \cdot r^2. \tag{3}$$

For a further explanation of the values used here we refer the reader to [2, 4] and the references therein. In our example the parameters for machine and tool are $k_{c1,1} = 2140\,(\frac{\text{N}}{\text{mm}^2})$ [6], $K_{st} = 1.3$, $m_v = 0.071$, $m = 0.25$ [6], $\kappa = 84^\circ$, $a_p = 3.05\,(\text{mm})$. Thus, (3) simplifies to

$$e_{cut} = \frac{8485.1}{1000} \pi \frac{\text{N}}{\text{mm}} \cdot \left(\frac{100 \frac{\text{m}}{\text{min}}}{v_c} \right)^{0.071} \cdot \left(\frac{1\,\text{mm}}{f \cdot \sin(84^\circ)} \right)^{0.25} \cdot r^2 \tag{4}$$

with restrictions $v_c \in [120, 190](\frac{\text{m}}{\text{min}})$ and $f \in [0.05, 0.12]\,(\text{mm})$ and radius $r = 30\,\text{mm}$ of the bar. We assume that there are no further restrictions and so the optimal value is attained for $v_c = 190\,(\frac{\text{m}}{\text{min}})$ and $f = 0.12\,(\text{mm})$ with $e_c^{opt} \approx 39000\,(\text{J})$.

The workpiece can be heated to the five different temperatures 800°C (arcs with index 1), 900°C (arcs with index 2), 1000°C (arcs with index 3), 1100°C (arcs with index 4), 1200°C (arcs with index 5). So we include five arcs $(n_2, n_3)^i$, $i = 1, \dots, 5$, one for each temperature. The forging and the deflashing process depend on the temperature of the input and on two further parameters that do not influence the next steps. For this reason we can determine the best parameter setting for fixed input temperature and it is sufficient to include five arcs $(n_3, n_4)^i$, $i = 1, \dots, 5$, resp. $(n_4, n_5)^i$, $i = 1, \dots, 5$, each. Finally, we add the arc (n_5, t).

Then the optimization model contains the variables $x_{(s,n_1)}$, $x_{(n_1,n_2)}$, $x_{(n_2,n_3)^i}$, $x_{(n_3,n_4)^i}$, $x_{(n_4,n_5)^i}$, $i = 1, \dots, 5$ and $x_{(n_5,t)}$. The flow conservation constraints for all nodes read:

$$x_{(s,n_1)} = -(-x_{(n_5,t)}) = 1 \tag{5}$$

$$x_{(n_1,n_2)} - x_{(s,n_1)} = \sum_{i=1}^{5} x_{(n_2,n_3)^i} - x_{(n_1,n_2)} = x_{(n_5,t)} - \sum_{i=1}^{5} x_{(n_4,n_5)^i} = 0 \tag{6}$$

$$\sum_{i=1}^{5} x_{(n_3,n_4)^i} - \sum_{i=1}^{5} x_{(n_2,n_3)^i} = \sum_{i=1}^{5} x_{(n_4,n_5)^i} - \sum_{i=1}^{5} x_{(n_3,n_4)^i} = 0 \tag{7}$$

The correct treatment of the temperatures is achieved by coupling constraints

$$x_{(n_2,n_3)^i} = x_{(n_3,n_4)^i} = x_{(n_4,n_5)^i}, \qquad i = 1, \dots, 5. \tag{8}$$

With the energy demands determined via simulation, calculation and approximations the objective function reads

$$\begin{aligned} & e_{bar} x_{(s,n_1)} + 39x_{(n_1,n_2)} + 2000x_{(n_2,n_3)^1} + 2350x_{(n_2,n_3)^2} + 2600x_{(n_2,n_3)^3} \\ & + 2900x_{(n_2,n_3)^4} + 3200x_{(n_2,n_3)^5} + 53.873x_{(n_3,n_4)^1} + 42.313x_{(n_3,n_4)^2} \\ & + 33.15x_{(n_3,n_4)^3} + 26.017x_{(n_3,n_4)^4} + 20.268x_{(n_3,n_4)^5} + 0.143x_{(n_4,n_5)^1} \\ & + 0.105x_{(n_4,n_5)^2} + 0.076x_{(n_4,n_5)^3} + 0.057x_{(n_4,n_5)^4} + 0.043x_{(n_4,n_5)^5} \end{aligned} \tag{9}$$

with e_{bar} the energy demand of the metal bar.

5 Robust process chain optimization

The coefficients in the objective function of the process chain model are uncertain due to several reasons. For example measurement errors may occur for some experimentally determined energy demands. For energy demands determined via simulation or calculations, the models used cannot take all relevant aspects into account in detail. Furthermore it might be necessary to obtain data estimates from comparable process variants because, *e. g.*, other machines should be used or previous tests were carried out for different parameter settings. Even if the parameter setting is the same, tolerances of the raw material or varying precision of the feedstock may lead to (slightly) differing energy demands. For this reason, the calculation of the exact optimal value with corresponding solution for given fixed energy demands should not be used as the only decision criterion. In fact, one is interested in robust solutions, *i. e.*, solutions that are good even if real data, that appears during production, varies in comparison to the initial data. So we want to adapt the process chain optimization model in such a way that robustness aspects can be respected.

Regarding the uncertainty, we assume that all given energy demands e_p, $p \in P$, and e_r, $r \in R$, are normally distributed with given expected value μ and covariance matrix Σ, *i. e.*, $e \sim \mathcal{N}(\mu, \Sigma)$. In general, Σ is not a diagonal matrix because some parts of the energy data are correlated, *e. g.*, because the same theoretical models, the same machines or the same measurement devices were used or there are dependencies between the single processes. Due to missing covariance information we will demonstrate the effect of different specially chosen covariance matrices for the optimal robust solutions. In this paper we add a so called *chance constraint* [7] to the original model. This constraint will ensure that in an optimal solution the expected total energy demand does not exceed a certain value (either a certain energy level is given or this value is minimized in the objective function) with high probability η, also called *confidence level*.

The objective function of (2) resp. (9) describes the energy consumption. We will concentrate on this energy demand of the process variants, but the approach can also be applied to the case with additional consideration of resources. Since the chosen temperature is the main influencing factor for the whole process chain we show the effect of different covariance matrices for the heating process and the forging process. In contrast to Section 4 we only use an input temperature of 800°C and a flash thickness of 5 mm but we vary the ram speed (as above 250 $\frac{\text{mm}}{\text{s}}$, 350 $\frac{\text{mm}}{\text{s}}$, 450 $\frac{\text{mm}}{\text{s}}$, 500 $\frac{\text{mm}}{\text{s}}$ and 530 $\frac{\text{mm}}{\text{s}}$) and concentrate on these two processes.

The reduced optimization problem with nominal energy demand $e = (e_1, e_2^1, \dots, e_2^5)$ $= (2000, 53.873, 56.25, 58.637, 59.57, 60.092)$ (in kJ) can be written as

$$\min\, e_1 + \sum_{i=1}^{5} e_2^i x_2^i \tag{10}$$

$$\text{subject to } \sum_{i=1}^{5} x_2^i = 1 \tag{11}$$

$$x_2^i \in \{0, 1\},\, i = 1, \dots, 5 \tag{12}$$

with e_1 the energy demand of the heating process, e_2^i the energy demand of the forging variant i, $i = 1, \dots, 5$, and a binary variable x_2^i for each variant that is one if and only if the variant is used in the process chain.

For the robust model we add a new variable $y \in \mathbb{R}_+$ and assume $e \sim \mathcal{N}(\mu, \Sigma)$. In this case, requiring the probability, that the total energy demand is not greater than y, to be greater or equal to η, *i. e.*,

$$\textbf{Prob}(\sum_{p \in P} \mu_p x_p - y \leq 0) \geq \eta \tag{13}$$

may be expressed as a convex second order cone constraint ($\eta \geq 0.5$) [8]

$$y - \sum_{p \in P} \mu_p x_p \geq \Phi^{-1}(\eta) \|\Sigma^{1/2} x\|_2 \tag{14}$$

with Φ representing the standard normal cumulative distribution function. Finally, the optimization problem reads $\min y$ subject to (11), (12) and (14).

We test this for probability $\eta = 0.95$ and three different covariance matrices $\Sigma_1, \dots, \Sigma_3$ resp. three different correlations. Only Σ_1 is a diagonal matrix. In Σ_2 and Σ_3 some off-diagonal entries are non-zero. In these, we assume that the heating and the forging variants influence each other.

$$\Sigma_1 = \begin{bmatrix} (\frac{1}{20}e_1)^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & (\frac{1}{10}e_2^1)^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & (\frac{1}{10}e_2^2)^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & (\frac{1}{10}e_2^3)^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & (\frac{1}{10}e_2^4)^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & (\frac{1}{10}e_2^5)^2 \end{bmatrix} \tag{15}$$

With $\sigma_{i,j} = \sqrt{\Sigma_1(i,i)\Sigma_1(j,j)}$, $i, j = 1, \dots, 6$, the other matrices read

$$\Sigma_2 = \begin{bmatrix} \sigma_{1,1} & 0 & -\frac{1}{2}\cdot\sigma_{1,3} & -\sigma_{1,4} & -\frac{1}{2}\cdot\sigma_{1,5} & 0 \\ 0 & \sigma_{2,2} & 0 & 0 & 0 & 0 \\ -\frac{1}{2}\cdot\sigma_{1,3} & 0 & \sigma_{3,3} & 0 & 0 & 0 \\ -\sigma_{1,4} & 0 & 0 & \sigma_{4,4} & 0 & 0 \\ -\frac{1}{2}\cdot\sigma_{1,5} & 0 & 0 & 0 & \sigma_{5,5} & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{6,6} \end{bmatrix} \tag{16}$$

$$\Sigma_3 = \begin{bmatrix} \sigma_{1,1} & \sigma_{1,2} & \frac{4}{5}\cdot\sigma_{1,3} & \frac{3}{5}\cdot\sigma_{1,4} & \frac{2}{5}\cdot\sigma_{1,5} & \frac{1}{5}\cdot\sigma_{1,6} \\ \sigma_{1,2} & \sigma_{2,2} & 0 & 0 & 0 & 0 \\ \frac{4}{5}\cdot\sigma_{1,3} & 0 & \sigma_{3,3} & 0 & 0 & 0 \\ \frac{3}{5}\cdot\sigma_{1,4} & 0 & 0 & \sigma_{4,4} & 0 & 0 \\ \frac{2}{5}\cdot\sigma_{1,5} & 0 & 0 & 0 & \sigma_{5,5} & 0 \\ \frac{1}{5}\cdot\sigma_{1,6} & 0 & 0 & 0 & 0 & \sigma_{6,6} \end{bmatrix} \tag{17}$$

With $\eta = 0.95$ the first variant ($x_2^1 = 1$) is optimal for Σ_1, the third variant ($x_2^3 = 1$) is optimal for Σ_3 and the fifth variant (x_2^5) is optimal for Σ_2. This shows that consideration of dependencies between processes resp. process variants is important for the determination of optimal process chains in production. So in future, a better knowledge of the uncertainties of the data is necessary.

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