Matched instances of Quantum Sat (QSat) Product state solutions of restrictions

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Abstract

Matched instances of the quantum satisfiability problem have the following property: They have a product state solution. This is a mere existential statement and the problem is to find such a solution efficiently. Recent work by Gharibian and coauthors has made first progress on this question: They give an efficient algorithm which works for instances whose interaction hypergraph is restricted in a certain way.
We continue this line of research and give two results: First, an efficient algorithm is presented which works when the constraints themselves are restricted (the interaction hypergraph is not restricted). The restriction is that each constraint has at most 2 additve terms. Second, over the field of real numbers the problem of solving matched instances of QSat by product state solutions becomes NP-hard.

1 Introduction

Generalities. The quantum complexity class QMA (Quantum Merlin Arthur) is defined analogously to NP: Given a quantum state, a quantum comupter can efficiently check (with sufficiently high probability) that this state solves a given instance of the problem considered. The theory of Hamiltonian Complexity, see for example [10] is currently being developed to deal with the relevant questions. Many complete problems for this class have been found, and one of them is QSat, the quantum satisfiability problem (in fact it is complete for a restriction QMA_1 where the check succeeds with probability 1.) QSat has been introduced by Bravyi in [7]. Note that problems in QMA seem much more difficult that those in NP,as the witnessing quantum state in general is a sum of exponentially many terms and thus cannot even be described classically in any efficient way.

We focus on QSat. In particular we are interested in efficiently solvable subcases. Recently remarkable results have been obtained in this area: Already Bravyi shows that the quantum analogue of 2-Sat is efficiently solvable. It took 10 years until it was noted that quantum 2-Sat can in fact be solved in linear time [8],[9], as classical 2-Sat. More recently Gharibian and coauthors [3] has looked at QSat instances where each QBit (corresponding to variables of the classical case) occurs at most twice and obtained an efficient algorithm (as classical,too). This paper contributes to the program to find more cases of QSat with an efficient algorithm.

Notation. An instance of the quantum satisfiability problem on n QBits $1, \ldots, n$ is a conjunction of constraints $C_1, \ldots, C_m$. A constraint C is a quantum state acting on k QBits $i_1, \ldots, i_k$ from these n QBits:

$$C = \sum_{b_1,\ldots,b_k} \alpha_{b_1\ldots b_k} |b_1 \ldots b_k >_{i_1\ldots i_k}$$

with b_i ranging over the two Bits $0, 1$. States $|QB_1 >, \ \ldots \ , |QB_n >$ with $|QB_i >= \overline{a_{i,0}}|0 > \ + \overline{a_{i,1}}|1 >$ (with $\overline{a}$ the complex conjugate of a) are a solution of C iff

$$\sum_{b_1,\ldots,b_k} \alpha_{b_1,\ldots,b_k} \cdot a_{i_1,b_1} \cdot \ldots \cdot a_{i_k,b_k} = 0.$$

That is the state $|QB_{i_1} > \otimes \cdots \otimes |QB_{i_k} >$ is orthogonal to the state C. We only consider product state solutions $|QB_1 > \otimes \cdots \otimes |QB_n >$. By the way this avoids the problem of general solutions which cannot be efficiently described. Note that constraints can still be entangled and thus really quantum. An instance $C_1, \ldots, C_m$ is satisfiable iff we have states $|QB_i >$ of our QBits which solve all C_j simultaneously. Base states of a QBit are $|0 >$ and $|1 >$ and correspond directly to classical boolean values.

For $C = |b_1 \ldots b_k >$ a base state constraint $|QB_{i_1} > , \ldots \ , |QB_{i_k} >$ are a solution iff at least one $|QB_{i_j}|$ is a base state with $|QB_{i_j} > = \ |\neg b_j >$. Interpreting a base state constraint, for example $|10 >$, as the clause $\neg x_1 \vee x_2$, we see that the classical satisfiability problem is included in QSat. Note that $x_1 = 1, x_2 = 0$ is the unique assignment falsifying the clause.

An instance of QSat is *matched* iff for each constraint we can pick one among the QBits on which it acts (we call it the matched QBit) such that no QBit is picked twice. Collecting the Qbits of each constraint in one set (hyperedge) a QSat instance induces a hypergraph (with multiple edges) on vertices $1, \dots, n$. We call this the interaction hypergraph. Thus an instance is matched iff its interaction hypergraph has an SDR (system of distinct representives.) Note that an SDR can be found effciently by bipartite matching techniques. The graph considered is: One side the vertices $1, \dots, n$, the other side the hyperedges and each hyperedge is adjacent to the vertices it consists of. Then an SDR is a matching in which each hyperedge occurs. For a matched instance of the classical satisfiability problem a solution can easily be found by assigning the matched variables the right truth value.

Motivation. In [1] (see also [2]) the following result is proven by non-constructive means (of basic Algebraic Geometry): Each matched instance of QSat has a product state solution. The motivating question for this work is obvious: Can we find such a solution efficiently ? Following common usage, [8] [9] [3], we disregard all questions of numerical precision here.

In seminal work on this problem Gharibian and coauthors [3] has given a positve answer to the question provided the interaction hypergraph has certain restricting properties (and the constraints are generic.)

Results and techniques. A constraint has l additive terms iff exactly l among the $\alpha_{b_1 \dots b_k}$ are non zero. An instance has l terms iff each of its constraints has at most l additive terms.

With $l = 1$ we only have base state constraints and therefore the classical satisfiability problem. Our first result concerns the natural next step, matched QSat instances with 2 terms.

Theorem 1 *The following problem has an efficient algorithm. Input: A matched QSat instance with 2 additive terms. Output: A product state solution to this instance.*

Example 1.1 *We observe that the case $l = 2$ cannot be directly reduced to a classical constraint satisfaction problem as the case $l = 1$. A constraint with 2 terms, for example, is $(|000 > + |111 >)_{i,j,k}$. A product state solution consisting of base states must have one Qbit among i, j, k $|0 >$ and another one $|1 >$. Thus we have a 2−colouring of the hyperedge $\{i, j, k\}$. A 2−colouring means that not all vertices of a hyperedge have the same colour. Even in a matched instance a solution cannot be* *found by simply assigning the matched vertex of each hyperedge the right colour (as in the case of classical Sat.)*

Consider the Fano Plane, see for example [3]. It is a $3-$uniform hypergraph (that is each edge consists of exactly 3 vertices) with the following properties: First, it has an SDR, second it is not $2-$colourable.

We consider the QSat instance with constraints $(|000> + , |111>)_{i,j,k}$. for each edge $\{i, j, k\}$ of the Fano plane. It is matched and therefore must have a product state solution. It cannot have one of base states only because it is not $2-$colourable. As the QSat instance has 2 additve terms our theorem applies.

The proof of Theorem **1** is based on a simple observation: Constructing a product state solution of the constraint $(|000> + |111>)_{i,j,k}$ for example, means looking for complex numbers a_i, b_i, c_i for $i = 0, 1$ such that $a_0 \cdot b_0 \cdot c_0 \;=\; -\, a_1 \cdot b_1 \cdot c_1$. Taking logarithms we reduce this to a linear equation. Thus for a QSat instance of m constraints we get a linear system of m equations in n variables. (Actually, we choose to introduce 2 linear systems one for the real and one for the imaginary part.) For a matched instance the system has at most n equations. If the system is solvable we have a solution of the instance.

But what, if the system is unsolvable? We will see that the system is non-homogeneous. In this case we resort to an old idea of Seymour from the theory of $2-$colourability of hypergraphs, see [4] (we learned if from [5].) We consider the homogeneous versions of the system (just setting each right-hand-side to 0.) As we have at most n equations this system must have a non-trivial solution now. This allows us to assign base states to some QBits in order to solve some constraints, and a smaller matched instance remains to be solved.

Clearly, this approach is limited to $l = 2$ because otherwise taking logarithms does not yield a linear system.

It seems difficult to find efficiently a product state solution to a matched instance in general. Thus we look for indicators of algoritmic hardness. We have a result in this direction, too.

Theorem 2 *The following problem is NP-hard. Input: A matched QSat instance over the real numbers. Output: A product state solution with real coefficients.*

This means in particular that matched QSat instances are not always solvable over the reals (this reflects the fact that the reals are not algebraically closed.)

2 Proof of Theorem 1

An algorithm for symmetric instances. A symmetric constraint C has the form

$$C \;=\; \alpha\cdot|b_1\dots b_k> \;+\beta\cdot|\neg b_1\dots\neg b_k> \quad \text{with} \;\; \alpha\neq 0 \text{ and } \beta\neq 0.$$

We normalize one of the coefficients α or β to 1. A symmetric instance consists of symmetric constraints.

A base state solution to a symmetric constraint corresponds to a solution in the sense of Not-all-equal Sat of (the clause corresponding to) $|b_1\dots b_k>$ (or $|\neg b_1\dots\neg b_k>$).

Definition 2.1 *Let* $C \;=\; (|b_1\dots b_k> \;+\; \alpha|\neg b_1\dots\neg b_k>)_{i_1\dots i_k}$ *with* $\alpha = r\cdot\exp(\mathbf{i}\psi),\;\; r>0$, *be a symmetric constraint acting on QBits* $i_1\dots i_k$ *from QBits* $1,\dots,n$. *Let* x_i *be a variable corresponding to QBit* i.

(a) The left-hand-side of C *is*

$$LHS(C) \;=\; \sum_{j,b_j=1} x_{i_j} \;-\; \sum_{j,b_j=0} x_{i_j}$$

with $j=1,\dots,k$. *For* $\alpha=1$ *this is not unique and we pick one of the two possibilties as* $LHS(C)$.

(b) The radius equation of C *is* $LHS(C) \;=\; \ln r$.

(c) The phase equation of C *is* $LHS(C) \;=\; \pi+\psi$.

Example 2.1 *We consider the symmetric constraint* $|110> \;+\; \alpha|001>$ *with* $\alpha = r\cdot\exp(\mathbf{i}\psi),\;\; r>0$, *acting on the first* 3 *QBits,* $1,2,3$, *for simplicity. The left-hand-side is* $x_1+x_2 \;-\; x_3$.

Product state solutions correspond to solutions of the equation

$$a_1\cdot b_1\cdot c_0 \;\; +\alpha\cdot a_0\cdot b_0\cdot c_1 \;=\; 0.$$

Making the ansatz that $a_0 = b_0 = c_0 = 1$ *and* $a_1, b_1, c_1 \neq 0$ *and decomposing*

$$a_1 = s_1 \cdot \exp(\mathbf{i}\phi_1),\;\; b_1 = s_2 \cdot \exp(\mathbf{i}\phi_2)\;,\; c_1 = s_3 \cdot \exp(\mathbf{i}\phi_3)\;\; \textit{with } s_i > 0$$

we get solutions to the equation above from solutions to

$$s_1 \cdot s_2 \;=\; r \cdot s_3 \;\; \textit{and} \;\; \exp(\mathbf{i}\phi_1) \cdot \exp(\mathbf{i}\phi_2) \;=\; -\; \exp(\mathbf{i}\psi) \cdot \exp(\mathbf{i}\phi_3).$$

Taking logarithms, solutions to the preceding equations can be obtained from real solutions to

$$\ln s_1 \;+\; \ln s_2 \;-\; \ln s_3 \;=\; \ln r \;\; \textit{and} \;\; \phi_1 \;+\; \phi_2 \;-\; \phi_3 \;=\; \pi \;+\; \psi.$$

Thus it is sufficient to solve the radius and phase equation.

Proposition 2.1 *Let* $C_1, \ldots, C_m$ *be a symmetric QSat instance over QBits* $1, \ldots, n$*. Let* $t_1, \ldots, t_n$ *solve the linear system of the radius equations of* $C_1, \ldots C_m$ *for* $x_1, \ldots, x_n$ *over the reals. Let* $\phi_1, \ldots, \phi_n$ *solve the system of the angle equations (over the reals).*

Then the states

$$|QB_i> = \; |0> \; + s_i \cdot \exp(\mathbf{i}\phi_i)|1> \quad \textit{with} \;\; s_i = \exp(t_i)$$

are a solution to $C_1, \ldots, C_m$

The proof of this proposition is by calculation along the lines of Example 2.1.

Example 2.2 *The symmetric instance* $|00> +|11>$, $|00> -|11>$, *with both constraints on the same two QBits, is matched. The left-hand-side can be picked as* $-x_1 - x_2$ *for both constraints. The right-hand-side of the radius equation is* 0 *in both cases (giving the radius of* $\exp(0) = 1$.) *But the right-hand-side of the phase equations is* π *for the first constraint and* 0 *(or* 2π*) for the second. The phase equations have no solution and Proposition 2.1 does not apply.*

However, we have a solution of base stats, one QBit must be $|0>$ *and the other one* $|1>$ *. Observe that the radius* $s_i > 0$ *in Proposition 2.1 and solutions with base states are not found.*

Proposition 2.2 *Let* $C_1, \ldots, C_m$ *be a symmetric instance of QSat over QBits* $1, \ldots, n$*. Let* $a_1, \ldots, a_n$ *be a non-trivial solution for* $x_1, \ldots, x_n$ *of the homogeneous linear system*

$$LHS(C_1) \;=\; 0, \;\; \ldots, \;\; LHS(C_m) \;=\; 0$$

over the reals. Let

$$|QB_i> = |0> \quad \textit{if } a_i < 0 \quad \textit{and} \quad |QB_i> = |1> \quad \textit{if } a_i > 0,$$

and let $|QB_i>$ *be an arbitrary state if* $a_i = 0$*.*

Then we have: The state $|QB_1> \otimes \ldots \otimes |QB_n>$ *is a solution to any constraint of the instance which acts on at least one QBit* i *with* $a_i \neq 0$*.*

Proof. Let $C = (|b_1 \ldots b_k> + \alpha|\neg b_1 \ldots \neg b_k>)_{i_1 \ldots i_k}$ be a constraint of the instance. We have $LHS(C) = \sum_{j,b_j=1} x_{i_j} - \sum_{j,b_j=0} x_{i_j}$. Any non-trivial solution to $LHS(C) = 0$ assigns at least two of the variables $\neq 0$.

Assume that x_{i_j} with $b_j = 1$ is assigned > 0 then another $x_{i_{j'}}$ with $b_{j'} = 1$ is assigned < 0 or one of the $x_{i_{j'}}$ with $b_{j'} = 0$ is assigned > 0 in order that the sum is equal to 0. Assume the first alternative applies.Then $|QB_j >= |1>$ and $|QB_{j'} >= |0>$. The term $|\neg b_1 \ldots \neg b_k>$ evaluates to 0 regardless of the states of the remaining QBits as $\neg b_j = 0$. The term $|b_1 \ldots b_k>$ always evaluates to 0 as $b_{j'} = 1$. Thus the claim holds for C.

Assume the second alternative applies. Then we have $|QB_j >= |QB_{j'} >= |1>$. The term $|b_1 \ldots b_k>$ evaluates to 0 because $b_{j'} = |0>$. The term $|\neg b_1 \ldots \neg b_k>$ evaluates to 0 as $\neg b_j = 0$.

The remaining cases are: First, an x_{i_j} with $b_j = 1$ is assigned < 0, and second, all x_{i_j} with $b_j = 1$ are assigned 0) . These cases are easily treated in the same way finishing the proof.

All this is subsumed in the following algorithm.

Algorithm 2.1 *Input: A matched symmetric instance of QSat on QBits* $1, \ldots .n$*. Output: A solution to this instance.*

Set $I :=$ *the input instance.*

1. *Set up the radius and phase linear system of* I*. If they both have a solution assign QBit* i *for* $1 \leq i \leq n$ *as prescribed in Proposition 2.1. End.*
2. *Obtain a non-trivial solution to the homogeneous system as in Proposition 2.2.*
   Assign base states to the Qbits as in Proposition 2.2.
   Leave the remaining Qbits unassigned.
3. *Set* $I :=$ *the constraints of* I *which act only on QBits which are not assigned.*

4. If I has no constraints then assign the Qbits unassigned by now arbitrarily. End.
5. Goto 1.

Concerning correctness: First I always consists of those constraints of the input which act only on unassigned QBits. As such I is a matched instance throughout. All constraints which act on variables assigned in 2. are solved regardless of the states the remaining QBits.
The linear system considered in 1., 2. is a mapping $\mathbf{R}^{n'} \longrightarrow \mathbf{R}^{m'}$ with n' standing for the number of unassigned variables and m' for the number of constraints acting only on these variables. Therefore $m' \leq n'$. If 1. does not apply we have that the kernel of the linear system is non-trivial. Therefore 2. applies and at least 1 constraint is solved by the assignment regardless how the remaining QBits are assigned. Thus I obtained in 3. has at least one constraint less. If 1. applies the instance I is solved by the assignment according Proposition 2.1.

Concerning running time: $O(n)-$times solving linear equations yields $O(n^4)$.

Reduction to symmetric instances. We need the following general transformation rule.

Assign $|QB_i>=|b>$. For each constraint which acts on QBit i and at least one additional QBit we do the following: We decompose it as

$$|b>_i \otimes |\phi_1> + |\neg b>_i \otimes |\phi_2> .$$

and substitute it with $|\phi_1>$. If the first term is not present ($|\phi_1>=0$) we simply delete the constraint. For this transfomation we have: If we have no constraint acting only on i then any solution of the transformed instance yields a solution to the original instance after appending $|QB_i>=|b>$.

We have a matched instance of QSat with two terms. To begin with, we get rid of non-symmetric constraints with two terms. Non-symmetric constraints with two terms can be written as

$$|b> \otimes(|b_1 \dots b_k> + \alpha|c_1 \dots c_k>) \quad , \alpha \neq 0.$$

We apply the following tranformation rules to the instance.

1. Elimination of the constraint

$$|b>_i \otimes(|b_1 \dots b_k> + \alpha|c_1 \dots c_k>).$$

where i is not the matched Qbit of the constraint. We substitute this constraint simply with

$$|b_1 \dots b_k > + \alpha |c_1 \dots c_k > .$$

Note that Qbit i may well occur in the new instance. The instance remains matched and any solution to the new instance is a solution to the original instance.

2. Elimination of the constraint

$$|b >_i \otimes (|b_1 \dots b_k > + \alpha |c_1 \dots c_k >)$$

where i is the matched QBit of the constraint. In this case we use the rule Assign $|QB_i >= |\neg b >$. First we observe that the only constraint acting only on i can be the constraint considered as each constraint acting on i must act on its matching QBit, too. The constraint considered becomes true under the assignment. As QBit i is not the matched QBit of any other constraint any solution to the transformed instance yields a solution to the original instance.

After appyling 1. as long as it applies and then 2. in the same way, we have a matched instance in which all constraints with 2 terms are symmetric. We still need to eliminate constraints with only 1 term. We apply the following transformation rule to the instance.

3. Elimination of constraints $|b_1 \dots b_k >_{i_1,\dots,i_k}$. Let i_j be the matched QBit, then: Assign $|QB_{i_j}| = |\neg b_{i_j} >$. The new instance is matched. Assign $|QB_{i_j} >= |\neg b_{i_j} >$ ensures that any solution to the new instance gives a solution to the original one.

We iterate 3. as long as we have constraints with 1 term only. Finally, this yields a symmetric instance with two terms or an instance without constraints. In the last case we assign the QBits not assigned by now arbitrarily and have a solution to the original instance.

3 Proof of Theorem 2

We give a translation of a classical 3−Sat formula into a matched instance. For this translation we need several instances of QSat as building blocks. First the instance of 2 constraints

$$|00>_{i,j}, \ \ |11>_{i,j} \ .$$

This is a matched instance (matching the first constraint to i and the other one to j) with the following property: A product state is a solution to this instance iff $|QB_i>=|b>$ and $|QB_j>=|\neg b>, b=0,1$. This instance allows to restrict attention to states which are base states and thus classical boolean values. (Note that this instance is solved by the state $|01>+|10>$, but this is not a product state and therefore is not of relevance here.)

The conjunction constraint:

$$|011> \ + |1> \otimes(|01> \ + |10> \ + |00>) \ \text{ on QBits } \ i,j,k.$$

This constraint computes the logical $\wedge$ in the following sense: If $|QB_j>=|b>$ and $|QB_k>=|c>$ where $b,c=0,1$ then $|QB_i>\otimes|QB_j>\otimes|QB_k>$ is a solution to this constraint iff $|QB_i>=|b\wedge c>$.

For disjunction the constraint:

$$|0>\otimes(|01> \ + |10> \ + |11>) \ + \ |100> \ \ \text{ on QBits } \ i,j,k.$$

For negation we get $|00>+|11>$. If a product state is a solution we have: If one of the QBits is a basis state, the other one must its negation. (We have inclded this only for didactical purposes, we do not need this constraint.) Note that the two constraints above $|00>, \ |11>$ enforce that both QBits are basis states.

These building blocks can be assembled to get a matched instance which determines the truth value of a $3-$Sat formula with boolean variables $x_1,\dots,x_n$ given the truth values of the variables.
Initialization instance:

$$(|00>, \ |11>)_{1,n+1}, \ (|00>, \ |11>)_{2,n+2}, \ \dots \ (|00>, \ |11>)_{n,2n}.$$

This is clearly matched. For any solution we have that QBits $1,\dots,n$ are basis states and QBits $n+1,\dots 2n$ the corresponding negations. Moreover any combination of basis states of $1,\dots,n$ can occur in a solution.

Given truth values for QBits $1,\dots,n$ we can calculate the truth value of a clause with two disjunction constraints. The first QBit (the result) of these constraints always is a new QBit. This new Qbit is the matched Qbit of the constraint.

After this we use $m-1$ conjunction constraints to calculate the conjunction of the values computed for the m clauses before. The first QBit of each conjunction constraint is a new Qbit. Again it is the matched QBit.

Clearly this instance is matched. Let r be the QBit which contains the final result. With the additional constraint $|0>_r$ we can ensure that $|QB_r>=|1>$. Thus any solution gives a satisfying assignment. But with this constraint the instance is not any more matched. (We see here that product state satisfiability of QSat instances with one more constraint than the number of variables is NP-hard.)

But, over the reals the test $|QB_r>=|1>$ is possible without violating the matching condition. The building block for this is:

$$(|00> \ + |11>)_{t,s} \ , \ (|01> \ - |10>)_{t,s}$$

on two QBits t, s (for test.) Clearly by itself this is matched. A product state solution is

$$(a_0|0> +a_1|1>) \ \otimes \ (b_0|0> +b_1|1>) \ \text{ with } \ a_0 = b_0 = 1, \ a_1 = b_1 = i$$

as $a_0 b_0 \ = \ 1$ and $a_1 b_1 \ = \ -1$ and $a_0 b_1 \ = \ a_1 b_0 \ = \ i$.

It is easy to see that we have no real product state solutions: First, each product state solution is such that a_0, a_1, b_0, b_1 must all be $\neq 0$. For if $a_0 = 0$, for example, we must have that $b_1 = 0$, in order to have a solution of the first constraint. This means $b_0 = 1$ and then the state does not solve the second constraint.

Now

$$a_0 b_0 \ + \ a_1 b_1 \ = \ 0 \ \text{ and } \ a_0 b_1 \ - \ a_1 b_0 \ = \ 0$$

implies $a_0 \ = \ - \ a_1 b_1 / b_0$ and then with the second equation $- \ a_1 b_1^2 \ - \ a_1 b_0^2 \ = \ 0$. This implies $- \ b_1^2 / b_0^2 \ = \ 1$ and $b_1 / b_0 \ = \ i$. Thus we have no solution of real product states for the building block. (Note that $|00> -|11>$ is a real solution, but non-product.)

Now we add to the instance the following two test constraints:

$$((|00> \ + |11>) \ \otimes |0>)_{t,s,r} \ , \ ((|01> \ - |10>) \ \otimes \ |0>)_{t,s,r}$$

with r the result QBit above and t, s new Qbits. Clearly the whole instance is still matched. A product state solution over the reals must have $|QB_r>=|1>$ to solve the test constraints. If $|QB_r>=|0>$ we must have a solution to $|00> \ + |11>$

, $|01> - |10>$ on QBits s, t. Thus QBits $1 \dots n$ of a solution must be a satisfying assignment of the original $3-$Sat instance.

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onsidering constraints $1|000\dots00> + < 111\dots11|$ and their product stae satisfiability we have a (quantum motivated) relaxation of hypergraph $2-$colouring, see example 1.1. There are other (quantum) ways to relax classical constraint satisfaction problems, see for examole [11]. One may speculate if there is any relationship. Matched instances of constraint satisfaction problems in general do not seem to have been systematically investigated, in particular from an algorithmic point of view, but see[6].

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